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Options of the 5 Platonic Polyhedrons

Platonic solids are fascinating precisely because—even though they're defined by fixed angular and symmetry constraints—the notion of "size" can be set in several different ways. In other words, when we talk about the "size" of a Platonic solid, we have options. Typically, one might choose one of the following measures as a scale parameter:

1. **Edge Length (a):**

This is the simplest and most natural parameter for each regular polyhedron. Once you set the edge length, every other length in the polyhedron is

determined by geometric ratios that come from its symmetry. In many textbooks and research contexts, all formulas for surface area, volume, and sphere radii are expressed in terms of the edge length.

2. **Circumscribed Sphere Radius (R):**

The circumscribed sphere is the smallest sphere that contains the entire polyhedron—its surface touches every vertex. Scaling the solid so that its vertices all lie on a sphere of a given radius is a common “size” criterion. In fact, if you want to compare different Platonic solids on an equal footing (for example, when designing models to fit into the same spherical container), you use their

formulas for \(R\) in terms of \(a\) to adjust the edge lengths accordingly.

3. **Inscribed Sphere Radius (r):**

This sphere touches each face of the polyhedron from the inside. It is sometimes seen as representing the “core” of the structure. If you prefer to scale the solids by ensuring that the largest possible sphere that fits inside them is the same for all, you’d use the formulas that express \(r\) in terms of \(a\).

4. **Midradius (or Face-Centered Radius):**

In some cases, one might refer to the distance from the center to the center of a face (or even to an edge's midpoint). This isn't as standard as the other three measures but can be useful for certain design or modeling applications.

Because these solids are all "similar" (their shapes remain unchanged when you uniformly scale them), you have complete freedom: for any given solid you can choose an arbitrary edge length, which then propagates into all derived measures. Conversely, if you fix one of the sphere radii—for instance, placing all five solids into the same sphere—the required edge lengths will differ by fixed ratios specific to each polyhedron.

To organize these ideas, here’s a summary table that expresses the key relationships for each Platonic solid with edge length \(a\):

| **Solid** | **Faces** | **Circumscribed Radius \
(R\)** | **Inscribed Radius \
(r\)** | **Volume \(V\)**
| **Surface Area \(A\)** |

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| **Tetrahedron** | 4 | \(R = \dfrac{a\sqrt{6}}
{4}\) | \(r = \dfrac{a\

sqrt{6}}{12}\) | \(V = \
dfrac{a^3}{6\sqrt{2}}\) | \(A = \
sqrt{3}\,a^2\) |

| **Cube** | 6 | \(R = \dfrac{a\sqrt{3}}
{2}\) | \(r = \dfrac{a}{2}\)
| \(V = a^3\) | \(A = 6a^2\)
|

| **Octahedron** | 8 | \(R = \dfrac{a}{\
sqrt{2}}\) | \(r = \
dfrac{a\sqrt{6}}{6}\) | \(V =
\dfrac{a^3\sqrt{2}}{3}\) | \(A = 2\
sqrt{3}\,a^2\) |

| **Icosahedron**| 20 | \(R = \dfrac{a\
sqrt{10+2\sqrt{5}}}{4}\) | \(r
= \dfrac{a\sqrt{3}(3+\sqrt{5})}{12}\)

| \(V = \dfrac{5(3+\sqrt{5})}{12}\,a^3\) | \(A = 5\sqrt{3}\,a^2\) |

| **Dodecahedron**| 12 | \(R = \dfrac{a\sqrt{3}\,(\sqrt{5}+1)}{4}\) | \(r = \dfrac{a}{20}\sqrt{250+110\sqrt{5}}\) *(a more complex formula)* | \(V = \dfrac{(15+7\sqrt{5})}{4}\,a^3\) | \(A = 3\sqrt{\,25+10\sqrt{5}\,}\;a^2\)* |

* Many texts express the formulas for the dodecahedron in various equivalent forms.

How to Choose a “Size” Parameter

- **Equal Edge Lengths:**

  If you simply choose the same edge length for every solid, you preserve a pure measure of linear scale. However, the volumes and the bounding spheres will then vary quite a bit.

- **Equal Circumscribed Spheres:**

  Frequently used in art or design, this approach adjusts each solid so that its vertices lie on a sphere of the same radius. For example, if you want all five solids to “fit” into the same spherical

frame, you would solve for the unique edge length a for each that gives you that R.

- **Equal Volumes or Surface Areas:**

  In more applied contexts—such as comparing structural properties or material uses—you might want the invariance of volume or surface area. In these cases, you’ll use the volume or area formulas to compute the correct edge length for each solid.

- **Duality Considerations:**

  An intriguing aspect is that some Platonic solids are duals: the cube is dual to the octahedron, and the dodecahedron to the icosahedron (with the

tetrahedron being self-dual). Under appropriate scaling, the inscribed sphere of one becomes the circumscribed sphere of its dual, a fact that can critically inform how you choose your “size” parameter.

Further Thoughts

This exploration isn’t just mathematical whimsy—it has deep implications in design, architecture, art, and even natural forms. For instance:

- **Artistic and Architectural Modeling:**

  When constructing physical models, choosing the size definition that best meets your need (are you bound by a particular spatial envelope? Do you want a model with uniform volume distribution?) will dictate which formula to start with.

- **Symbolic and Aesthetic Meanings:**

  In many cultures, the inscribed sphere can represent the “inner perfection” of an object, while the circumscribed sphere symbolizes its outer limit. Comparing these ratios across the Platonic solids can yield interesting insights into their inherent beauty—a factor that has captivated architects and artists from the Renaissance to modern design.

- **Scaling and Proportions in Nature:**

  Although Platonic solids are idealized forms, their proportional relationships echo in natural structures, from molecules and crystals to large-scale architectural patterns. Exploring the “size” options provides a framework to understand how nature might favor one property (volume, surface area, or boundary constraints) over another.

By delving into these different “size” options, you gain a multiperspective view of these age-old shapes. Whether you fix an edge length, equalize a sphere’s radius, or set volume/surface area criteria,

each choice reveals a different aspect of the rich geometric tapestry of the Platonic solids.